

# Distinguishability of particles and its implications for peculiar mass transport in inhomogeneous media

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A mass transfer directed from low to high density region in an inhomogeneous medium is modeled as a limiting case of a two-component lattice gas with excluded volume constraint and one of the components fixed. In the long-wavelength approximation, density relaxation of mobile particles is governed by a diffusion process and interaction with a medium inhomogeneity represented by a static component distribution. It is shown that density relaxation can be locally accompanied by a density distribution compression. In quasi one-dimensional case, the compression dynamics manifests itself in a hopping-like motion of diffusing substance packet front position due to a staged passing through inhomogeneity barriers and leads to a fragmentation of a packet and retardation of its spreading. A root-mean-square displacement reflects only an averaged packet front dynamics and becomes inappropriate as a transport characteristic in this regime. In a stationary case mass transport throughout a whole system may be directed from a boundary with low concentration towards a boundary with that of high one. Implications of the excluded volume constraint and particles distinguishability for these effects are discussed.

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## I. INTRODUCTION

Mass transfer in inhomogeneous media and complex systems is often governed by nonclassical diffusion laws and exhibits peculiar transport effects. Specific examples include sub- or superdiffusion processes of, for example, Brownian particles in hydrodynamic flow [1–4], and absolute negative mobility effect [5–13]. Very often, studying such effects we encounter a situation where relaxation goes in counterintuitive way, namely, instead of expected spreading density distribution undergoes compression, at least over certain time period, or even full collapse. In other words, mass transport is directed from low to high density region. In particular, such behavior occurs in systems with absolute negative mobility or negative diffusion coefficient.

The phenomenon of absolute negative mobility or negative diffusion coefficient may be of different origin. In particular, it can occur due to time- or particle-particle correlations, or nonequilibrium character of a process. This effect was demonstrated, for example, in electron-hole plasma, as an electron-drag effect arising from an electron-hole scattering [5], in models of interacting Brownian particles [11–13] or interacting lattice gas [14], and for nonequilibrium dynamics of Brownian particle in a periodic one-dimensional potential or a quasi one-dimensional channel in a presence of external force periodic in time [7–9]. Note that the negative sign of diffusion coefficient entails absolute instability in a system and may signify the onset of a new phase formation, e.g., nuclei growth at first order phase transition [14, 15] or collapse of electron-hole plasma that may be a precursor

for a neutral exciton formation. [16].

The phenomenon of mass transfer from low to high density region also may be caused by an interaction of particles with medium inhomogeneity. In this case density relaxation through this interaction may go much faster than via diffusion process, and, as a result, the latter may not be a main relaxation and entropy production mechanism [16]. A simple, although speculative, example of such a density relaxation is a non-wettability effect, caused by interaction between a liquid drop and a substrate (medium). Analogous effect in diffusive systems will be of primary interest in this paper.

Here we demonstrate that such an effect may naturally appear for diffusing particles in inhomogeneous media. In this case density relaxation dynamics in a subdiffusive regime can be locally accompanied by compression of density distribution. Moreover, in a stationary case, mass transport through the inhomogeneous sample can go from a boundary with low concentration to a boundary with that of high one.

In order to demonstrate these effects we resort to the simplest model of a two-component lattice gas assuming that every lattice site can be occupied by one particle only (excluded volume constraint). In the limiting case of a frozen or static component this model reduces to the mass transport of mobile component in the inhomogeneous medium. Another reason to use such a model is an appearance of a drag effects, which has been found in this framework in a presence of driving field and nonzero particle hopping rates of both components, see, e.g., [17, 18]. In addition, absolute negative mobility effect may appear in a case of non-Markov dynamics [10]. Two component lattice gas may also display memory effects. For example, correlation between jumps of tagged particle and its influence on tracer diffusion was examined in [19, 20]. This correlation represents the tendency of a tagged par-

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ticle to return to its previous position and it follows from: (i) a presence of different particle sort, i.e. particle distinguishability, and (ii) the excluded volume constraint. The latter corresponds to infinite repulsion energy between particles at the same lattice site. Formally, such “back correlations” describe a repulsion of a tagged atom from the other gas component.

## II. THE MODEL

We consider the simplest model of a two-component lattice gas on a regular lattice, see, e.g., [19]. It is assumed that every lattice site can be occupied by a single particle of a sort  $m$  or  $n$  only. Particles can hop between neighboring sites separated by a distance  $a$  with rates  $\nu_m$  and  $\nu_n$ , respectively. Hopping events are assumed to be instantaneous, i.e., particle spends most of the time being localized on a lattice site. Rate equations for occupation numbers can be written as [19]

$$\frac{d\hat{m}_i}{d\tau} = \nu_m \sum_j [\hat{m}_j(1 - \hat{m}_i - \hat{n}_i) - \hat{m}_i(1 - \hat{m}_j - \hat{n}_j)], \quad (1a)$$

$$\frac{d\hat{n}_i}{d\tau} = \nu_n \sum_j [\hat{n}_j(1 - \hat{n}_i - \hat{m}_i) - \hat{n}_i(1 - \hat{n}_j - \hat{m}_j)]. \quad (1b)$$

Here  $\hat{m}_i = \{0, 1\}$  and  $\hat{n}_i = \{0, 1\}$  are local occupation numbers of  $m$  and  $n$  particles' sort at  $i$ th site, respectively, and sum is taken over nearest neighbors of  $i$ th site only. Equations (1) are based on the assumption that it is possible to neglect fluctuations in the number of jumps between sites  $i$  and  $j$  [21], and terms of the type  $\nu_m \hat{m}_j(1 - \hat{m}_i - \hat{n}_i)d\tau$  gives a mean number of jumps (of  $m$ -particle from site  $j$  to site  $i$  per time  $d\tau$  in this particular case).

In what follows we shall restrict our study to a macroscopic dynamics. The simplest, if somewhat rough, way to obtain evolution equations for the average local occupation numbers  $m_i = \langle \hat{m}_i \rangle$  ( $\langle \dots \rangle$  being statistical average) is to apply the mean field approximation [22] or the local equilibrium approximation [21]. The latter corresponds to the introduction of a local chemical potential associated with a given lattice site, or a coarsened Zubarev statistical operator [23]. In this approach we lose all the information on fast (as compared to local equilibration time) processes and neglect any correlations.

Next, we apply the long-wavelength approximation assuming lattice constant  $a$  to be much smaller than characteristic inhomogeneity length scales  $l_m$  and  $l_n$  of  $m$  and  $n$  components, respectively,  $l_m \sim l_n \gg a$ . In the continuum limit, equations of motion for the densities take the form

$$\frac{1}{\nu_m} \frac{dm}{d\tau} = \nabla[\nabla m - (n\nabla m - m\nabla n)], \quad (2a)$$

$$\frac{1}{\nu_n} \frac{dn}{d\tau} = \nabla[\nabla n - (m\nabla n - n\nabla m)], \quad (2b)$$

where we have introduced dimensionless coordinate  $r/a$ . Obtained equations describe smooth density profiles and can not be applied for length scales which are comparable with a lattice constant,  $l_m \sim l_n \sim a$ , where short-range correlations become significant, as near percolation threshold, see [19]. Note that in the case where inhomogeneity length scales of two components drastically differ, say  $l_m \gg l_n \gg a$ , higher orders of space derivatives should be taken into account in Eqs. (2).

More general form of Eqs. (2) including terms taking into account external driving field have been obtained using mean field approximation [22] or phenomenological approach [17, 24], and exploited for investigation of phase transitions [17, 22, 24], drifting spatial structures [25], and unusual transport effects in two-component driven diffusive systems [18, 25].

The main difference of Eqs. (2) from the ordinary diffusion equation is a presence of mixing flux  $\mathbf{j}_{mn} = n\nabla m - m\nabla n = -\mathbf{j}_{nm}$ . This flux describes mutual drag of particles of one sort by particles of another one. It is caused by the two same reasons as a “back correlations” [19], mentioned above. The first one is a distinguishability of two different sorts of particles, e.g., by spin or color. The second one is a local interaction (repulsion) between particles created by excluded volume constraint.

The flows of gas components  $\mathbf{j}_m \equiv -\nu_m \nabla m + \nu_m(n\nabla m - m\nabla n)$  and  $\mathbf{j}_n \equiv -\nu_n \nabla n + \nu_n(m\nabla n - n\nabla m)$  can be represented as sums of diffusive and hydrodynamic parts. For example, for  $m$ -component,  $\mathbf{j}_m = \mathbf{j}_m^d + \mathbf{j}_m^h$ , the flow  $\mathbf{j}_m^d = -\nu_m(1 - n)\nabla m \equiv -D_m \nabla m$  describes diffusion of  $m$ -particles through vacant sites, unoccupied by  $n$ -particles, with local diffusion coefficient  $D_m \equiv \nu_m(1 - n)$ . Term  $\mathbf{j}_m^h = m(-\nu_m \nabla n) \equiv m\mathbf{V}_m$  may be associated with transfer of  $m$ -particles by some hydrodynamic flow with velocity  $\mathbf{V}_m = -\nu_m \nabla n$ , where concentration  $n$  of another component plays a role of a velocity potential.  $\mathbf{j}_n$  can be analyzed along the same line.

The drag of particles of one sort by particles of another sort directly follows from Eqs. (2). Consider the particular case where characteristic inhomogeneity length scale of one component is much smaller than that of another component, say,  $l_m \gg l_n \gg a$ . To a first approximation, currents of both components are governed by a concentration gradient of the single component,  $\nabla n$ , at least for a certain period of time:

$$\frac{1}{\nu_m} \frac{dm}{d\tau} \approx \nabla(m\nabla n), \quad \frac{1}{\nu_n} \frac{dn}{d\tau} \approx \nabla[(1 - m)\nabla n]. \quad (3)$$

The drag effect particularly means that mass transfer of  $m$ -particles can be directed along their concentration gradient,  $\nabla m$ , i.e., from region with low concentration to region with that of high one.

We will be interested in consequences of the drag effect in the limiting case where hopping rate of  $n$ -particles is negligibly small in comparison with  $m$ -ones,  $\nu_n \ll \nu_m$ . It is intuitively clear that this is some approximation of mass transport in an inhomogeneous medium where diffusion of mobile  $m$ -particles occurs through vacant sites

unoccupied by “heavy”  $n$ -particles. Considering component  $n$  as a static one, we can write reduced equation of motion for a density of mobile component  $m$

$$\frac{dm}{dt} = \nabla[(1-n)\nabla m + m\nabla n] = (1-n)\nabla^2 m + m\nabla^2 n, \quad (4)$$

where  $t = \nu_m \tau$ . Equation (4) describes advection-diffusion with compressible flow ( $\nabla^2 n \neq 0$ ). Equation of this type is often used for the description of a transport in various systems [1, 2]. In a compressible flow, transport may exhibit intriguing effects as it is in the presence of stable foci (traps) of the flow [1]. In our case, the hydrodynamic flow,  $\sim m\nabla n$ , is associated with a repulsion of mobile particles from the frozen component. Thus, density relaxation of mobile particles is governed by both diffusive mechanism and their interaction with “internal” field  $\nabla n$ .

### III. INFLUENCE OF AN INTERNAL MEDIUM FIELD

As it is well known, the advection-diffusion can exhibit anomalous behavior that manifests itself, in particular, in non-classical dependence of a root-mean-square displacement on time,  $R = \langle r^2 \rangle^{1/2} \sim t^\zeta$ , where  $\zeta \neq 1/2$  is the exponent of anomalous diffusion [1]. In the case of compressible flow considered here ( $\nabla^2 n \neq 0$ ), the exponent  $\zeta$  may lose universality and may depend on the degree of compressibility. Such anomalous phenomenon is not necessarily takes place asymptotically in the limit  $t \rightarrow \infty$ , but can occur, at least, on a time scale that is less than a finite mixing time [1, 4]. On this time scale, density relaxation may strongly depend on initial state and demonstrate peculiar behavior.

In this section we show that mass transport from low density region towards a dense one in an inhomogeneous medium may locally accompany subdiffusive ( $\zeta < 1/2$ ) process. Moreover, such integral characteristic as a root-mean-square displacement  $R$  does not describe relaxation process properly. Unlike in the case of ordinary diffusion, motion of packet front  $r_f(t)$  does not coincide with a root-mean-square displacement time dependence and has phased behavior.

Indeed, at relatively low values of gradient  $\nabla m$  mass transfer is determined by the second term in right hand side of Eq. (4),  $\dot{m} \approx m\nabla^2 n$ , which, at least over short time period, leads to the dependence

$$m(r, t) \approx m(r, 0) \exp(t\nabla^2 n(r)). \quad (5)$$

Behavior of a density profile  $m(r, t)$  in the vicinity of frozen component distribution  $n(r)$  local maxima and minima is different. Near minima, where  $n(r_{min} + \delta r) \approx n_{min}(1 + q^2(\delta r)^2)$ , mobile particles tend to accumulate, i.e., initial density profile is squeezing,

$$m(r, t) \approx m(r, 0) \exp(tq^2 n_{min}), \quad (6)$$

which means that mass transfer may occur towards higher concentration region. Contrary to that, mobile particles tend to evacuate from regions close to maxima, where  $n(r_{max} + \delta r) \approx n_{max}(1 - q^2(\delta r)^2)$ , which can be interpreted as forcing mobile component out by the frozen one

$$m(r, t) \approx m(r, 0) \exp(-tq^2 n_{max}). \quad (7)$$

Such exponential dynamics indicates a presence of the faster process than that of diffusive, which locally, over short time periods, leads to the dependence  $R_{loc} \sim t$ , while in the case of anomalous diffusion  $R \sim t^\zeta$ , with  $\zeta < 1$ . In this regard, such global integral characteristic as root-mean-square displacement

$$R(t) = \sqrt{\langle x^2 \rangle} = \left( \frac{\int x^2 m(x, t) dx}{\int m(x, t) dx} \right)^{1/2} \quad (8)$$

may be improper for the description of a diffusion process due to loss of information on fast local dynamics.

#### A. Packet fragmentation and hoping dynamics of its front position

In order to illustrate features of the transport phenomenon in an inhomogeneous medium described by Eq. (4), we consider relaxation process in quasi one-dimensional case, supposing that transverse size  $L_\perp$  of a system is of the order of magnitude or less than characteristic inhomogeneity length scale of the densities  $m$  and  $n$ ,  $L_\perp \leq l_m \sim l_n$ . Then equation (4) reduces to

$$\dot{m} = (1-n)\partial_x^2 m + m\partial_x^2 n, \quad (9)$$

where  $\partial_x$  labels one-dimensional derivative.

We will consider spreading of initial Gaussian distribution

$$m(x, 0) = M \exp(-x^2/4l^2) \quad (10)$$

in a periodic field of the frozen component

$$n(x) = (N/2)(1 - \cos k_0 x). \quad (11)$$

Degree of medium inhomogeneity is determined by the amplitude  $N$  and period  $2\pi k_0^{-1}$  of the frozen component.

As it can be seen from Fig. 1(a), presence of inhomogeneity slows down packet spreading. Such slowing down may be caused by decreasing of effective diffusion coefficient (renormalization of time) and/or decreasing of index  $\zeta$ , which characterizes subdiffusive regime (see inset at Fig. 1(a)). However, similar behavior of index  $\zeta$  (see inset at Fig. 1(b)) is given by Eq. (9) with only diffusion term included  $\dot{m} = \partial_x[(1-n)\partial_x m]$ , where relaxation has another character, Fig. 1(b). Packet fragmentation is associated with the second term in Eq. (4), which describes repulsion of light atoms from heavy ones. If this term is dominant, mass transfer does not go through a diffusive

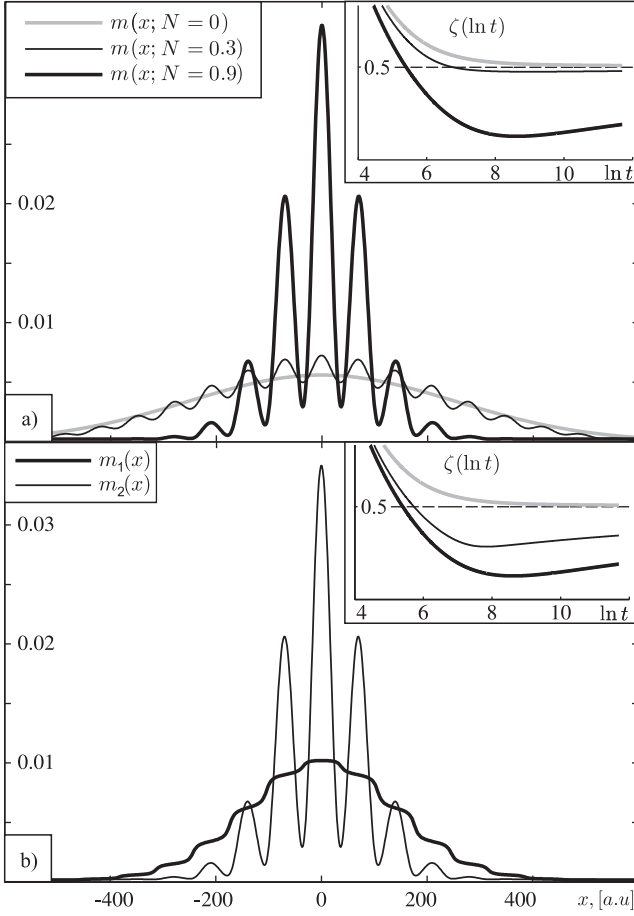


FIG. 1: a) Density distribution  $m(x, t)$  for different values of  $N$  at  $t = 31888$ ; b)  $m_1(x)$  is the density distribution, given by equation  $\dot{m}_1 = \partial_x[(1-n)\partial_x m_1]$ ,  $m_2(x)$  obeys Eq. (9). Insets display corresponding index behavior  $\zeta(\ln t) = \ln R(t)/\ln t$ , where  $R(t)$  is given by Eq. (8). Initial amplitude  $M = 0.1$ , initial half-width  $\sqrt{2}l = 14.14$ , and  $k_0 = 0.1$  are used in both (a) and (b) cases.

mechanism, described by the first term in right hand side of Eq. (4), but instead occurs due to interaction with a heavy subsystem.

As it directly follows from Eq. (4), local flow  $\mathbf{j}_m$  of the mobile component is defined by two contributions,

$$\mathbf{j}_m = -(1-n)\partial_x m - m\partial_x n. \quad (12)$$

The first term,  $\mathbf{j}_m^d = -(1-n)\partial_x m \sim -(1-n)(m/l_m)$ , where  $l_m$  is typical length scale of  $m(x, t)$  variation near  $x$ , describes standard diffuse spreading, i.e., tendency of transfer to be directed towards concentration lowering. Presence of the frozen component leads to a decrease of diffusion coefficient by the factor of  $1-n$ . Frozen component plays the role of a barrier for mobile particles. As concentration  $n(x)$  (barrier height) increases the velocity of penetration through the barrier by mobile atoms (diffusive flux  $\mathbf{j}_m^d$ ) decrease as  $1-n$ . The flux  $\mathbf{j}_m^h = -m\partial_x n \sim -m(nk_0)$  describes repulsion of mobile atoms from frozen one. Fluxes  $\mathbf{j}_m^d$  and  $\mathbf{j}_m^h$ , defined by a

density gradients  $\partial_x m$  and  $\partial_x n$ , may have different signs and behave as two competing fluxes. If  $1/n - 1 < l_m k_0$ , then total flux  $\mathbf{j}_m$  may be locally directed towards region with higher concentration, which means local compression of mobile atom distribution.

Figure 2 illustrates dynamics of initial Gaussian distribution spreading, which has phased nature. Front propagation through the first barrier towards neighboring local profile minimum (depicted at Fig. 2(a)) consists of two stages. At first, particles locally accumulate in the minimum due to  $\mathbf{j}_m^h$  domination until the condition  $2/N - 1 > l_m k_0$  for  $m(x, t)$  profile is met, or in other words, until local “pressure” ( $\propto \partial_x m$ ) gets high enough to overcome repulsion from next barrier. Expression  $2/N - 1 > l_m k_0$  defines the condition for diffusive penetration through the barrier. Then process repeats for the next local minimum, while in the previous one particles undergo usual spreading, Fig. 2(b). Such staged process describes motion of a packet front  $x_f(t)$ , which position is determined by condition  $\dot{m}(x_f, t) = 0$ , see Fig. 2(c).

Front  $x_f(t)$  divides  $x$ -axis in two regions: (i)  $|x| < x_f$ , where concentration  $m(x, t)$  is decreasing for any  $x$  and density relaxation is going mainly due to diffusion with diffusion coefficient renormalized by a profile of the frozen component; (ii)  $|x| > x_f$ , where concentration is increasing and relaxation is strongly affected by a repulsion from the frozen component (medium inhomogeneity). The latter leads to a local compression and density increase.

As can be seen from Fig. 2(c), medium inhomogeneity, which in our case is determined by an amplitude  $N$ , leads to appearing of difference between packet’s front motion  $x_f(t)$  and root-mean-square displacement  $R(t)$  dynamics. Front  $x_f(t)$  exhibits phased behavior and is governed by two linear in time, fast and slow processes. Packet front spends most of the time being localized inside a barrier and performs quick jumps into neighboring barriers while root-mean-square displacement (Eq. (8)) gives only an averaged dynamics of front motion, see Fig. 2(c).

## B. Compressibility of initial density distribution

Compression (growth) of density distribution also can occur for the central packet during certain initial time interval. Indeed, from Eqs. (4), (10), and (11) it is easy to estimate behavior of distribution amplitude at the peak over short time periods,

$$m(0, t) \approx M + \frac{Mt}{2l^2}(Nk_0^2 l^2 - 1). \quad (13)$$

Amplitude  $m(0, t)$  tends to increase if  $N > (k_0 l)^{-2}$ . On the other hand, density distribution compression means that velocity (time derivative) of a mean-square displacement  $v(t) = d\langle x^2 \rangle / dt$  is negative. The latter may be roughly estimated in Fourier domain

$$v(t) = -\frac{1}{m_0} \partial_k^2 \dot{m}_k \big|_{k=0}, \quad (14)$$

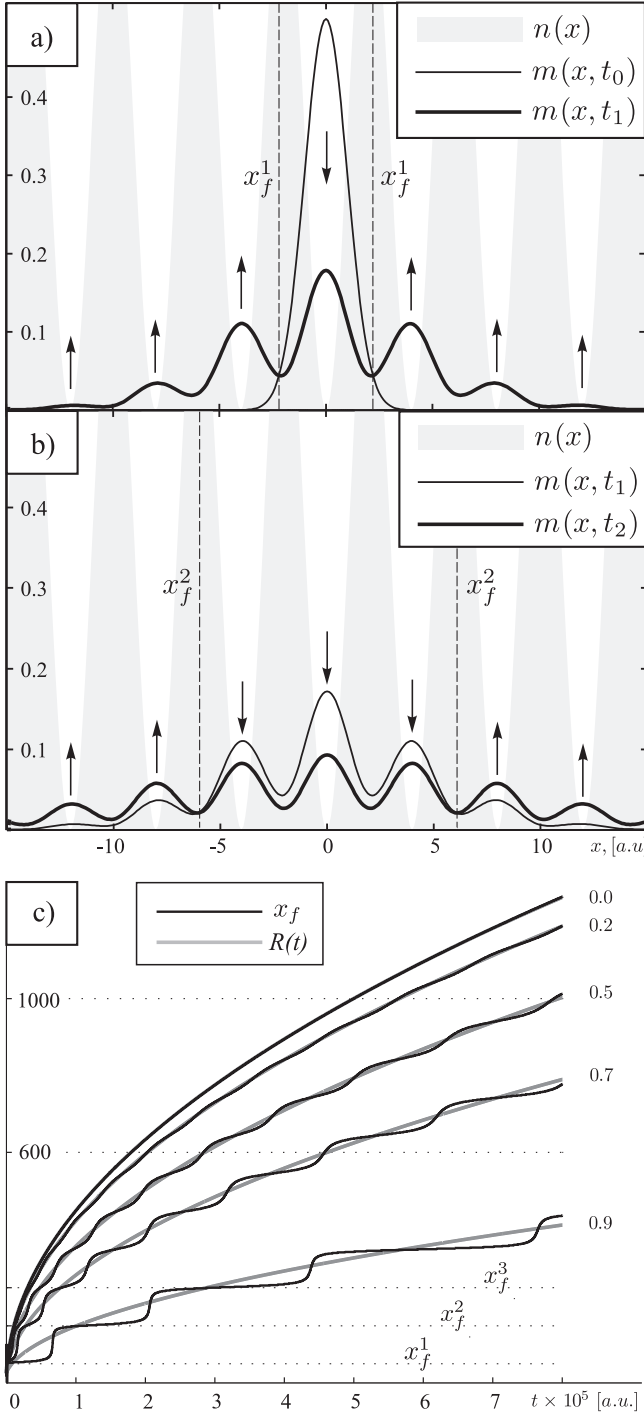


FIG. 2: Packet dynamics during time intervals: a)  $[t_0, t_1]$ , b)  $[t_1, t_2]$ ; ( $t_0 = 0$ ,  $t_1 = 14 \times 10^3$ ,  $t_2 = 45 \times 10^3$  time units). Initial amplitude  $M = 0.5$ , amplitude of the frozen component  $N = 0.7$ . Shaded pattern shows density distribution profile of the frozen component  $n(x)$ , given by Eq. (11), arrows denote whether sub-packet is squeezing (growing) ( $\uparrow$ ) or spreading ( $\downarrow$ ) on certain time interval. Dashed lines  $x_f^i$  denote region of main localization during corresponding time interval. c) Time dependence of packet front  $x_f(t)$  defined as a point where  $\dot{m} = 0$ , and a root-mean-square displacement  $R(t)$  defined by Eq. (8), for different values of  $N$ . Lines' labels correspond to values of  $N$ . Dotted lines  $x_f^i$  correspond to the dotted lines at Figs. 2 (a) and (b).

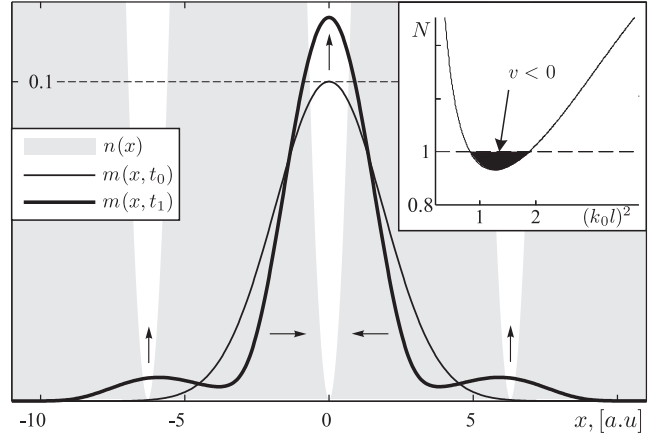


FIG. 3: Central peak squeezing (growth) of a mobile component density profile during initial time interval  $[t_0, t_1]$ , ( $t_0 = 0$ ,  $t_1 = 3 \times 10^4$  time units). Initial amplitude  $M = 0.1$ ,  $N = 0.9$ , initial half-width  $\sqrt{2}l = 1.7$ , and  $k_0 = 1.05$ . Shaded pattern shows density distribution profile of the frozen component  $n(x)$ , given by Eq. (11), arrows indicate directions of packet evolution. Filled region at the inset is determined by inequality (19) and condition  $N < 1$ , and corresponds to a negative velocity of packet motion.

where  $m_k(t)$  is the Fourier image of  $m(x, t)$  and obeys equation

$$\dot{m}_k = -(N/4)[(k^2 - 2kk_0)m_{k-k_0}] - (N/4)[(k^2 + 2kk_0)m_{k+k_0}] - k^2[1 - (N/2)]m_k. \quad (15)$$

Since  $v(t)$  is determined by limit  $k \rightarrow 0$ , it is reasonable to approximate Eq. (15) by its expansion for small  $k$  ( $k \ll k_0$ )

$$\dot{m}_k \approx -k^2[1 - (N/2)]m_k - (N/2)k^2m_{k_0} - Nk^2k_0 \frac{\partial}{\partial k_0}m_{k_0} - (1/4)Nk^4 \frac{\partial^2}{\partial k_0^2}m_{k_0}, \quad (16)$$

where we have omitted terms with  $m_{2k_0} \approx 0$  assuming that typical scale of inhomogeneity for  $m(x, t)$  is of the order or less than that of for  $n(x)$ , i.e.,  $k \leq k_0$ . Taking limit  $k \rightarrow 0$  we obtain expression for velocity

$$v \approx (2 - N) - \frac{N}{m_0} \left( 1 + 2k_0 \frac{\partial}{\partial k_0} \right) m_{k_0}. \quad (17)$$

Here  $m_{k_0}$  is defined by the equation

$$\dot{m}_{k_0} \approx -k_0^2 \left( 1 - \frac{N}{2} \right) m_{k_0} + \frac{N}{4} m_0. \quad (18)$$

From (17) condition for packet squeezing ( $v < 0$ ) directly follows, see inset at Fig. 3,

$$2/N < 1 + (4k_0^2 l^2 - 1) \exp(-k_0^2 l^2). \quad (19)$$

Numerical solution of Eq. (9) corresponding to such a case is shown at Figure 3. Initial compression lasts for

certain time period after which diffusion goes by means of sequential overcoming of potential barriers (as it was discussed in previous subsection). Note that the estimate (19) is rough and initial packet squeezing actually appear at smaller values of amplitude  $N$ , as shown at Fig. 3.

#### IV. THE STATIONARY CASE

As it was shown in the previous section, interaction of diffusing particles with inhomogeneities of a medium leads to a local particle accumulation, i.e., local mass transport towards higher concentration. It is interesting that such transport effect can be realized at global scale, throughout a whole system, with arbitrary distribution  $n(x)$ . In order to illustrate such a possibility we consider quasi one-dimensional boundary problem

$$(1 - n)\partial_x^2 m + m\partial_x^2 n = 0 \quad (20)$$

with boundary conditions

$$m(0) = m(0), \quad m(L) = m(L), \quad (21)$$

where  $L$  is a sample length. Solution of Eq. (20) has the form

$$m(x) = (1 - n(x)) \left( \frac{m(0)}{1 - n(0)} - J \int_0^x \frac{d\xi}{(1 - n(\xi))^2} \right), \quad (22)$$

where  $J$  is a total particle flux through the system

$$J = - \left( \int_0^L \frac{d\xi}{[1 - n(\xi)]^2} \right)^{-1} \left( \frac{m(L)}{1 - n(L)} - \frac{m(0)}{1 - n(0)} \right) = \frac{1}{L} \int_0^L \mathbf{j}_m(x) dx. \quad (23)$$

In the last expression  $\mathbf{j}_m(x)$  is given by Eq. (12).

As we see, stationary density distribution  $m(x)$  may be inhomogeneous even if concentrations at the boundaries  $m(L) = m(0)$  are equal.

As it was mentioned above, mass transfer in such a system is governed not only by the mean field of  $m(x)$  gradient, but also by interaction of a mobile subsystem with a frozen one. As it follows directly from Eq. (12) mass transfer may go from lower concentrations towards higher one. Assuming  $m(L) > m(0)$  one gets, that flux  $J$  is directed towards the higher concentration region, i.e., from the boundary  $x = 0$  to the boundary  $x = L$ , if the following condition is satisfied

$$\frac{m(L)}{m(0)} < \frac{1 - n(L)}{1 - n(0)}. \quad (24)$$

Thus total flux direction is determined by boundary conditions for  $m$  and  $n$  components.

#### V. SUMMARY AND DISCUSSION

The presence of a second sort of particles in lattice gas leads to peculiar transport effects. In the case of a two-component gas mass transport is affected by an action of an additional flux, contrary to the case of a single component gas of indistinguishable particles. This flux is associated with mixing of different gas components (interdiffusion term) and in a presence of interaction between particles such as excluded volume constraint in our case, and may lead to a drag of one sort of particles by another one. If particles of one sort have negligibly small mobility in comparison with another sort, so that one sort is assumed to be static, then the mixing flux, or its part, of mobile particles transforms into a stationary hydrodynamic flow that may drag mobile particles. As a result, density relaxation of mobile particles occurs via diffusion process and/or via an interaction with a medium, i.e., with a frozen gas component.

In our paper we have presented a case of a system where diffusive mechanism cooperates with density relaxation under action of “internal medium field”, given by a periodic distribution of a static component. Such mutual cooperation leads to unusual transport with local mass transfer resulting in an increase of concentration.

Indeed, as it was shown, if medium is strongly inhomogeneous, its “field” leads to a local accumulation of diffusing particles in minima of inhomogeneity profile. Presence of such a local processes is manifested by fragmentation of a packet during its spreading. The dynamics of a diffusive process also becomes peculiar and mass transport occurs as sequential penetration through inhomogeneity barriers, i.e. regions with high concentration of the static component. This leads to a phased character of a packet front motion that consists of a sequence of fast and slow processes (moves) with linear in time coordinate dependence. Such front behavior is similar to motion of a defect (slow motion inside barrier and fast passing through inhomogeneity minima). Contrary to the case of ordinary diffusion behavior of a root-mean-square displacement  $R(t)$  and that of a packet front  $x_f(t)$  do not coincide.  $R(t)$  reflects only an averaged packet front dynamics. Local accumulation of particles, which accompanies diffusive process and sequential barriers overcoming, also slows down packet spreading. The latter entails subdiffusive regime. In stationary case interaction of diffusing particles with medium inhomogeneity can lead to mass transport directed from a boundary with low concentration to a boundary with that of high one.

Note, that quasi one-dimensional case considered in this work is rather illustrative, because dimensionality reduction leads to enhancement of particle-particle correlations. In two- and three-dimensional cases demonstrated effects may not be so pronounced. However, accounting for interparticle interaction on nearest neighboring sites leads to essential changes in mass transport [14] and may enhance discussed effects.

Note, that assigning appropriate mobilities to compo-

nents in Eqs. (2) we can model not only inhomogeneous media but also systems with quenched randomness, as it was pointed out in [17, 24], or, for example, quenched inhomogeneities, dynamically generated in glasses [26].

Distinguishability of two sorts of particles and excluded volume constraint are responsible for unusual transport effects appearing in two-component lattice gas. Note, these effects may speculatively be considered as a purely statistical consequence of particles' distinguishability in a multicomponent non-interacting Fermi gas.

Equations (2) have been obtained using rough approximations, which entails certain limitations on their validity. First of all we neglect fast processes in the system, as it is generally the case for the diffusion approximation, the framework basic Eqs. (1) are written in. In addition, Langevin source of fluctuations in a number of jumps between lattice sites was omitted in Eqs. (1). We also lose information on fast processes applying the mean-field approximation which is equivalent to the local equilibrium one.

This approximation also means that we neglect any correlations in the system, in particular, short-range "back correlations" (local memory effect) [19, 20] that are known to contribute to effective diffusive process. The long-wavelength approximation does not allow to consider mass transport on length scales comparable with the lattice distance. In particular, it is in a vicinity of percolation threshold where short-range correlations become significant.

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